Notes on theory errors for the one-jet inclusive cross section

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We discuss the correlated systematic errors that may be ascribed to the next-to-leading order QCD theory used to predict the one-jet inclusive cross section in hadron collisions.

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Contents

I. INTRODUCTION

Predictions of the Standard Model are typically made with the aid of next-to-leading order (NLO) perturbative calculations (or sometimes with NNLO calculations). Evidently, these predictions are not exactly equal to what one should measure if the Standard Model is correct. If we have an NLO calculation, we leave out NNLO and N^{3}LO contributions, *etc.* We also leave out contributions that are suppressed by a power of the large momentum scale of the problem. Of course, we do not know exactly how big these contributions are: if we could calculate them, we would include them in the prediction. Nevertheless, we can estimate the size of the corrections. They then constitute a “theory error” in the prediction, which is quite similar to an experimental systematic error in the measurement.

In this paper we distinguish between errors associated with higher order contributions and power suppressed contributions to the cross section, which we call theory errors, and errors associated with our imperfect knowledge of the parton distribution functions needed for the prediction.

Estimated theory errors are needed in two contexts. First, if an experiment does not agree with the theoretical prediction within the experimental statistical and systematic errors, then we need to see if there is agreement within the combined experimental and theory errors and the errors from the parton distributions used in the prediction. In the case that the disagreement is outside of the combined errors, then we have a signal for new physics.

The second context in which we need estimated theory errors is in the determination of parton distribution functions from experimental measurements. The theory errors give a contribution to the errors that we associate with the parton distribution functions that emerge from a fit to the data. Evidently, if we do not include theory errors, the resulting errors in the parton distribution functions will be too small. Additionally, if for one kind of process the theory error is large while for another kind of process the theory error is small, then we will give the large-error process too much weight in the fit.

In this paper, we provide an estimate of the theory error for the one jet inclusive cross section $d\sigma/dE_T$, where E_T is the transverse momentum or “transverse energy” of the jet, in hadron-hadron collisions. There is good data for this process from the CDF and D0 experiments at Fermilab, including careful estimates of the experimental systematic errors. Estimates of the theory error are needed to accompany the estimates of the experimental systematic errors.

We warn that there is no unique method to estimate theory errors. Thus our task is to provide a method that is defensible if not necessarily optimal. We seek to provide an estimate in a form that includes the correlations from one E_T to another.

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II. GENERAL SETUP

We use next-to-leading order (NLO) QCD theory to make predictions for the one-jet inclusive cross section[1]

$$\frac{d\sigma}{dE_T} = \frac{1}{2(y_{\max} - y_{\min})} \times \\ \times \left[\int_{-y_{\max}}^{-y_{\min}} dy + \int_{y_{\min}}^{y_{\max}} dy\right] \frac{d\sigma}{dE_T\,dy} \quad . \tag{1}$$

In the calculation, one uses Monte Carlo integration. Then there is a random statistical error for each point $E_{T,i}$. We do not include these statistical errors in the analysis here since they are typically quite small (say 2%) and one can reduce them by running the program for a longer time. If we wished to include the errors from fluctuations in the Monte Carlo integrations, that task would be straightforward because the statistical nature of these fluctuations is known.

The treatment of theory errors beyond the errors from fluctuations in the Monte Carlo integrations is more subtle. These are similar to the correlated systematic errors in the experimental results. We formulate a treatment the theory errors as follows. We let

$$\frac{d\sigma}{dE_T} = \left[\frac{d\sigma}{dE_T}\right]_{\rm NLO} \left\{1 + \sum_i \lambda_i f_i(E_T)\right\} \quad . \tag{2}$$

Here the functions $f_i(E_T)$ are definite functions, while the λ_i are unknown parameters. Thus $\lambda_i f_i(E_T)$ represents an unknown theoretical contribution that might modify the NLO theory. We treat the λ_i as Gaussian random variables with variance 1. That is, the size of the uncertainty with label i is represented by how big $f_i(E_T)$ is. Note that the uncertainty is a function of E_T that is expressed in terms of uncertain parameters, the λ_i. If we were to believe that the uncertainty is of order, say, 10% but we have no idea of what the shape of the true cross section is within a 10% band about the NLO prediction, then we would choose many functions $f_i(E_T)$, each of size 0.1, but with each being non-zero only in a very tiny range of E_T. Such a view seems to us unreasonable. Experience with various perturbative and non-perturbative contributions teaches that they are smooth functions of the relevant variables, E_T in this case. On the other hand, experience also teaches that new effects have some dependence on the variables at hand. For instance, some non-perturbative contributions may be much larger (as a fraction of the NLO cross section) at low E_T than at high E_T. Some perturbative contributions beyond NLO may be larger at high E_T than at low E_T. Thus we seek a few functions $f_i(E_T)$ that have some dependence on E_T and represent, as best we can determine, our understanding of the character of uncalculated contributions.

In the following sections, we analyze several sources of theory errors and associate them with functions $f_i(E_T)$.

[1] Specifically, we use the program of [1], although there are other programs that can give the same results.

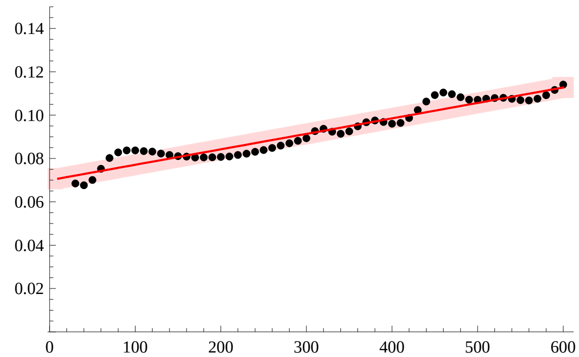

Figure 1: The largest eigenvalue λ_{max} of the matrix M plotted versus E_T in GeV. This represents approximately a $2\lambda_{max} = 14\%$ uncertainty at $E_T = 50$ GeV and an $2\lambda_{max} = 23\%$ uncertainty at $E_T = 600$ GeV. The shaded (red) band is a straight line fit.

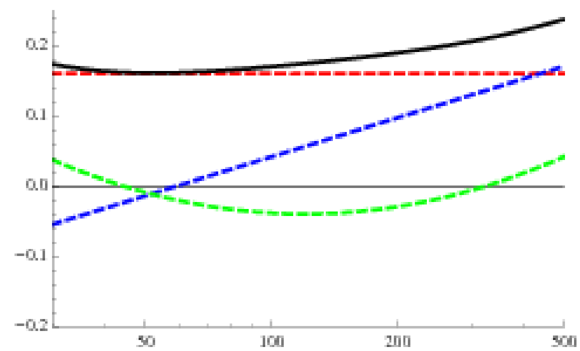

Figure 2: The net error as given by Eq. (6). Also shown, as dashed lines, are the individual functions $f_i(E_T)$.

III. ERROR ESTIMATE FROM SCALE DEPENDENCE

We can follow a version of the traditional estimation of theory errors from the dependence of the computed NLO cross section on two scales, the renormalization scale $\mu_{\rm uv}$ and the factorization scale $\mu_{\rm co}$. Our implementation is as follows. Let us denote

$$\begin{aligned} x_1 &= \log_2(2\mu_{\rm uv}/E_T) \ , \\ x_2 &= \log_2(2\mu_{\rm co}/E_T) \ . \end{aligned} \tag{3}$$

We compute the cross section near $x_1 = x_2 = 0$ and fit it to the form

$$\left[\frac{d\sigma(x_1,x_2)}{dE_T}\right]_{\rm NLO} = \left[\frac{d\sigma(0,0)}{dE_T}\right]_{\rm NLO} \times$$
$$\times \left[1 + \sum_J x_J A_J + \sum_{J,K} x_J M_{JK} x_K\right] \quad . \quad (4)$$

The vector A_J tells where the cross section has a saddle point as a function of the scales. The saddle point is generally a slightly smaller than at $\mu_{\rm co} = \mu_{\rm co} = E_T/2$, that is at slightly negative values of x_1 and x_2. We return to this point in the following section. For the moment, we consider the matrix M.

We use M to provide an estimate of uncalculated higher order terms, on the ground that the order α_s^2 contribution to M from the scale dependence of the coupling and the parton distributions is canceled by NNLO contributions to the hard scattering cross section. We compute the eigenvalues of M and denote by $\lambda_{\rm max}$ the one with the larger absolute value. We find (see Fig. 1) that $|\lambda_{\rm max}|$ is about 0.08 at low values of E_T, between 30 GeV and 100 GeV and rises a bit to about 0.11 at higher values of E_T, around 500 GeV. Traditionally, one estimates the theory error by making an excursion away from the "best" scale choice of a factor 2 for each of the scales, that is ± 1 for each of x_1 and x_2. That is, one uses an excursion of size 2 in $x_1^2 + x_2^2$. This suggests that the "traditional style" error estimate should be $2|\lambda_{\rm max}|$. That is, the error varies between about 0.16 at lower values of E_T to about 0.22 at higher values of E_T.

We can choose functions $f_i(E_T)$ that give this, approximately. We choose

$$\begin{aligned} f_0(E_T) &= 0.16 \quad , \\ f_1(E_T) &= 0.08 \left[\ln(15\, E_T/\sqrt{s}) + 0.7\right] \quad , \\ f_2(E_T) &= 0.04 \left[\ln(15\, E_T/\sqrt{s})^2 - 1.0\right] \quad . \end{aligned} \qquad (5)$$

With these choices, the net error,

$$\mathcal{E}(E_T) \equiv \sqrt{\sum f_i(E_T)^2} \quad , \qquad (6)$$

is a slowly rising function of E_T that is about 0.2. See Fig. 2.

Comment on the range of scale choices. We have estimated the theoretical uncertainty by varying the μ scales by a factor of two about a central value, which we have chosen as the saddle point in the dependence of the cross section on the scales. This is a conventional choice, but is it reasonable? To examine this question, we can look at cases in which NNLO calculations exist. Here, we choose one case as an example. In Fig. 3, we show the NNLO calculation for Higgs production at the LHC as a function of a parameter $p_T^{\rm veto}$ [2]. Here, the renormalization and factorization scales are varied by a factor of two,

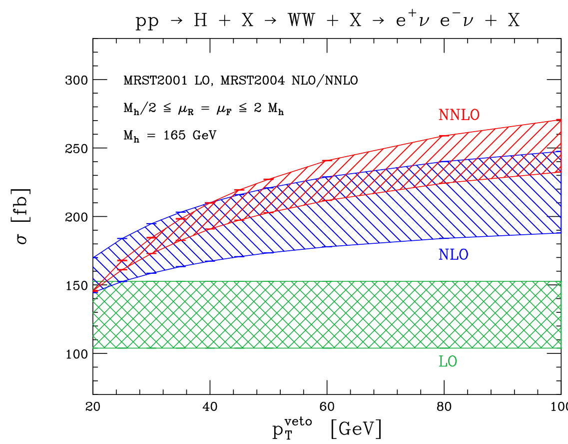

Figure 3: The cross section for Higgs production at the LHC for LO, NLO, and NNLO calculations as taken from Ref. [2]. The computed cross section vetos jets in the central region $|\eta| < 2.5$ imposes $p_T^{jet} > p_T^{veto}$.

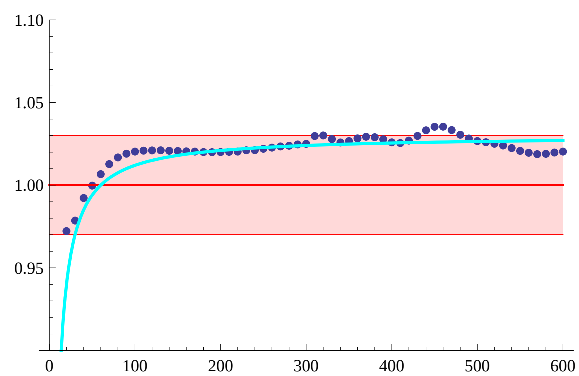

Figure 4: Fractional shift of the jet cross section at the saddle point compared to the nominal choice $\{\mu_{IR}, \mu_{CO}\} = \{E_T/2, E_T/2\}$. The shift is generally within a $\pm 3\%$ band (indicated by the shaded region) except in the low E_T range. The curve is the function $f_3(E_T) - f_4(E_T)$, where f_3 and f_4 are given in Eq. (7).

$\{\mu_R, \mu_F\} \in [M_h/2, 2M_h]$. We find that this range of uncertainty covers the full NNLO range for a low $p_T^{\rm veto}$ and includes the NNLO central value at large $p_T^{\rm veto}$. We conclude that in this case varying the parameters μ by a factor of 2 in the NLO calculation provides reasonable estimate of the "1σ" error in the NLO calculation. Choosing a smaller range would underestimate the error.

IV. UNCERTAINTY DUE TO CHOICE OF $\mu = E_T/2$

For the calculation of jet cross sections, it is convenient to choose the renormalization scales such that $\{\mu_{IR}, \mu_{CO}\} = \{E_T/2, E_T/2\}$ since this point is generally near the saddle point of the NLO cross section as a function of $\{\mu_{IR}, \mu_{CO}\}$. We can evaluate the value of the jet cross section at both $\{\mu_{IR}, \mu_{CO}\} = \{E_T/2, E_T/2\}$ and at the saddle point. The difference yields an estimate of the theoretical uncertainties that is independent of the uncertainty derived from the quadratic terms, $\sum x_J M_{JK} x_K$, in Eq. (4). In Fig. 4, we display the fractional shift of the jet cross section at the saddle point compared the cross section at $\{\mu_{IR}, \mu_{CO}\} = \{E_T/2, E_T/2\}$. This shift is approximately a constant 2% to 3% for E_T above 100 Gev. For low E_T, the shift appears to have another contribution that is approximately $10^{-3}/(E_T/\sqrt{s})$. We treat both of these shifts as independent contributions to the theory error. Thus we add two error functions $f_i(E_T)$,

$$
\begin{aligned}
f_3(E_T) &= 0.03 \quad . \\
f_4(E_T) &= \frac{0.001}{E_T/\sqrt{s}} \quad .
\end{aligned}
\tag{7}
$$

V. SUMMATION OF THRESHOLD LOGS

For parton-parton scattering near the threshold for the production of a jet with a given E_T, there is restricted phase space for real gluon emission. Thus, there is an incomplete cancellation of infrared divergences between real and virtual graphs, resulting in large logarithms. At n-th order in $\alpha_S^n(\mu)$ these logarithms enter the cross section in the form $\alpha_S^n(\mu)\, L^{2n}$. The singular logarithm L is of the form

$$
L^n \simeq \left[\frac{\ln^{n-1}\left((\hat{s}-4E_T^2)/E_T^2\right)}{\hat{s}-4E_T^2}\right]_+
$$

where $\hat{s}$ square of the energy available in the partonic c.m. frame.[2] Under the integral the function with the "+-prescription" is multiplied by a smooth function $F(s)$ that contains the parton distribution functions in the initial state hadrons. We can express this as

$$
\begin{aligned}
&\int d\hat{s} \left[\frac{\ln^{2n-1}\left((\hat{s}-4E_T^2)/E_T^2\right)}{\hat{s}-4E_T^2}\right]_+ \mathcal{F}(\hat{s}) \\
&= \int d\hat{s}\, \ln^{2n-1}\left(\hat{s}-4E_T^2\right) \frac{\mathcal{F}(\hat{s}) - \mathcal{F}\left(4E_T^2\right)}{\hat{s}-4E_T^2} \\
&\sim \int d\hat{s}\, \ln^{2n-1}\left(\hat{s}-4E_T^2\right) \mathcal{F}'\left(4E_T^2\right) \quad .
\end{aligned}
$$

Thus, the +-prescription has the effect of taking the derivative of $F(\hat{s})$, denoted $F'(\hat{s})$. These contributions are potentially large if the parton distribution

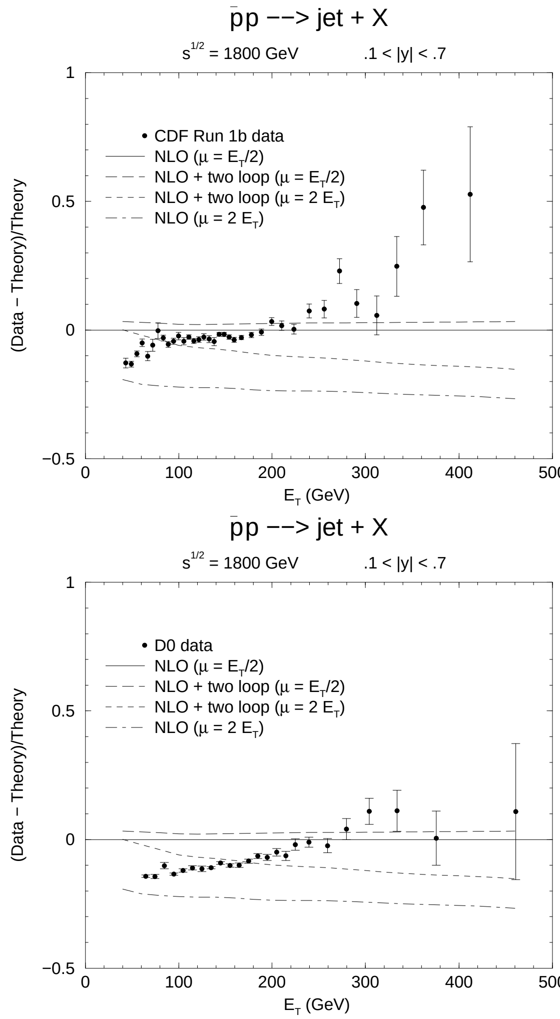

Figure 5: The effect of the threshold resummation contributions to jet production at the Tevatron, $\sqrt{s} = 1800$ GeV for two choices of scale, $\mu = \{E_T/2, 2E_T\}$. Figure a) compares with the CDF data, and Figure b) compares with the D0 data. These figures are from Ref. [3].

functions contained in $F(\hat{s})$ are steeply falling. The leading logarithms can be summed to all orders in α_S. We make use of the numerical results from Ref. [3].

Figure 5 displays the size of the threshold corrections for CDF and D0 jet measurements at the Tevatron. Pairs of curves are presented for two choices of scale: $\mu = E_T/2$ and $\mu = 2E_T$. For $\mu = 2E_T$, the effects of the threshold resummation can be sizable, 10% at high E_T and up to 20% at low E_T. However, for a scale choice of $\mu = E_T/2$, the threshold summation effects are uniformly small, approximately 2% throughout the kinematic range.

We use the threshold summation as an independent error, associating with it another function

[2] See Ref. [3] for details.

$f_i(E_T)$ that we take to be

$$f_5(E_T) = 0.02 \quad . \tag{8}$$

VI. UNDERLYING EVENT & HADRONIC CORRECTIONS: SPLASH-IN AND SPLASH-OUT

A separate source of uncertainties in jet measurements comes from what is colloquially known as "splash-in" and "splash-out" corrections. "Splash-in" corrections arise from the underlying event, which can deposit additional energy into the jet cone; we will refer to these more formally as Underlying Event (UE) corrections. "Splash-out" corrections come from the hadronization process of the jet which may move some of the jet energy outside the defined jet cone. We will refer to these as Hadronization Corrections (HC). In either case, the correction is modeled as adding an amount δE_T to the observed transverse energy (or transverse momentum) of the jet. To estimate δE_T, follow the analysis of Ref. [4].

We can parameterize the effect of the underlying event corrections on the apparent E_T of the jet as

$$\langle \delta E_T \rangle_{UE} = \Lambda_{UE}\, R^2 \tag{9}$$

where R is the cone radius of the jet, and Λ_{UE} is a parameter we will extract from comparative studies. Because we model the "splash-in" energy is random and uncorrelated how the jet develops, we expect the contribution from the underlying event will scale as the area of the jet cone–hence the factor of R^2. Ref. [4] finds $\Lambda_{UE}(1.96 \text{ TeV}) \simeq 3 \pm 1$ GeV.

In contrast to the underlying event correction, we expect the "splash-out" correction to increase as the size of the jet cone decreases. Following Ref. [4], we parameterize the hadronization correction as

$$\langle \delta E_T^i \rangle_{HC} = -C_i\, \frac{2}{R}\, \mathcal{A}(\mu_{\mathcal{I}}) \tag{10}$$

where $\mathcal{A}(\mu_I)$ parameterizes the soft gluon radiation. Ref. [4] takes $\mu_I = 2$ GeV and finds $\mathcal{A}(0.2 \text{ GeV}) \simeq 0.2$ GeV. In Eq. (10), C_i is a color factor which depends on whether the jet is initiated by a quark or a gluon. For gluons we use $C_A = 3$, and for quarks we use $C_F = 4/3$. Note the ratio $C_A/C_F = 9/4 \sim 2$ implies that the gluon jets have broader hadronization radiation than the quark jets.

The results for the underlying event and hadronization corrections are displayed in Figs. 6 and 7. Fig. 6, taken from Ref. [4], displays the corrections for quark jets at the Tevatron using a seedless cone algorithm. Fig. 7 displays the corrections for both quark and gluon jets at the Tevatron using the parameterizations of Eqs. (9) and (10). Fig. 6 corresponds to Fig. 7-a) where we have used our parameterizations of Eqs. (9) and (10). The underlying event and hadronization corrections have opposite sign, and we note that for a jet cone radius of R between 0.6 and 0.7, the two corrections nearly cancel each other.

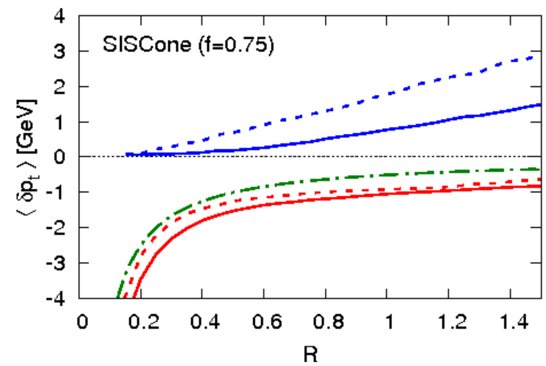

Figure 6: Modification of the p_t of jets due to the underlying event (upper curves) and hadronization (lower curves), for $qq \to qq$ scattering at the Tevatron Run II ($p\bar{p}$, $\sqrt{s} = 1.96$ TeV) as computed in Ref. [4]. The individual curves compare `Pythia` 6.412 [5] tune A [dashed line], and `Herwig` 6.510 [6] with `Jimmy` 4.3 [7] [solid line]. The analytical result for the hadronization correction as represented by Eq. (10) is also displayed [dot-dashed line]. The figure is from Ref. [4].

While the underlying event correction is the same for quark and gluon jets, the hadronization correction depends on the appropriate color factor, C_F or C_A. While we cannot easily distinguish quark or gluon jets on an event-by-event basis, we can estimate the average fraction of each. Fig. 8 displays the relative fraction of qq, qg, and gg initiated jets at the Tevatron as a function of E_T for the leading-order processes. Using this as a guide, we estimate that the fractions of final-state quark and gluon jets at the Tevatron in the low E_T region is approximately

$$\begin{aligned} f_q &\approx \frac{2}{3} \\ f_g &\approx \frac{1}{3} \quad . \end{aligned}$$

Using these fractions, we can form a weighted average of the quark and gluon terms

$$\begin{aligned} [\delta E_T]_{HC} &= f_q [\delta E_T^q]_{HC} + f_g [\delta E_T^g]_{HC} \\ &= -\; f_q \frac{2C_q}{R} \mathcal{A}(\mu_I) - f_g \frac{2C_g}{R} \mathcal{A}(\mu_I) \\ &\simeq -1\,\text{GeV} \pm 0.5\,\text{GeV} \quad . \end{aligned}$$

Here, we have used a typical cone radius of $R = 0.7$ and taken a conservative choice for the uncertainty of 50% of the correction.

The underlying event corrections are determined

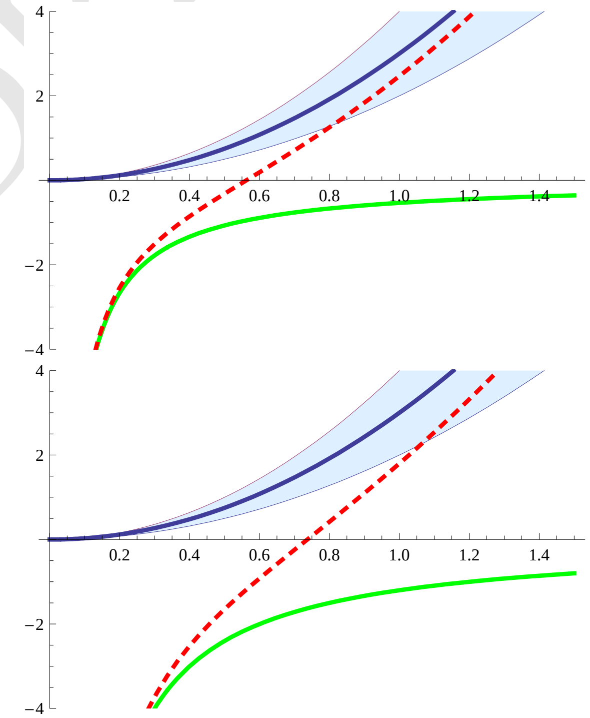

Figure 7: Calculation of the underlying event and hadronization corrections for a) quark-initiated and b) gluon-initiated jets at the Tevatron. The shaded (blue) band represents the underlying event correction, with its uncertainty, based on $\Lambda_{UE} = 3 \pm 1$. The lower solid (green) line represents the hadronization correction. The sum of these corrections is represented by the dashed (red) line.

by $\Lambda_{UE} = 3 \pm 1$ GeV; thus,

$$[\delta E_T]_{UE} \simeq +1.5\,\mathrm{GeV} \pm 0.5\,\mathrm{GeV} \quad .$$

Combining the underlying event and hadronization corrections, we obtain

$$\delta E_T \simeq +0.5\,\mathrm{GeV} \pm 0.7\,\mathrm{GeV} \quad , \tag{11}$$

where we have added the separate uncertainties in quadrature.

Using this δE_T shift, we can determine the effect on the final jet observables once we know the behavior of the jet cross section as a function of E_T. The differential cross section obeys an approximate power law of the form

$$\frac{d\sigma(E_T)}{dE_T} \approx \frac{const}{E_T^n} \quad . \tag{12}$$

For jets in the intermediate range $E_T = [50, 300]$ GeV, we find $n \approx 7$ as illustrated by Fig. 9 and Fig. 10.

The effect of the non-perturbative corrections is to shift the apparent E_T via

$$E_T = E_T^{\mathrm{pert}} + \delta E_T \quad .$$

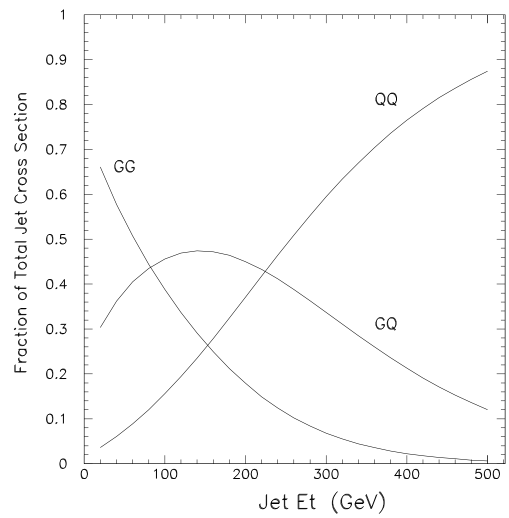

Figure 8: Relative contribution to the inclusive jet cross-section due to the various partonic subprocesses. From Ref. [8].

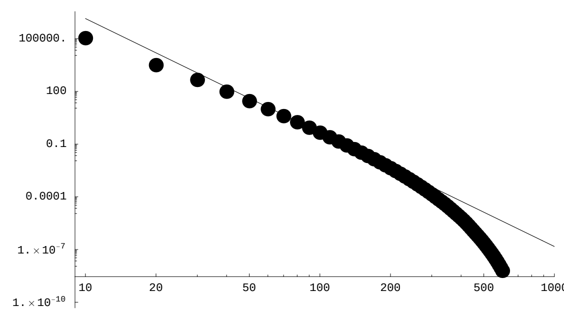

Figure 9: Jet Cross section vs. E_T at the Tevatron. The line is a power law fit with $n = 7$; this describes the slope of the jet data in the intermediate range $E_T = [50, 300]$ GeV.

Let us write the differential cross section as the function f:

$$\frac{d\sigma(E_T)}{dE_T} \equiv f(E_T)$$

Then

$$f(E_T) \approx f_{\mathrm{pert}}(E_T^{\mathrm{pert}}) = f_{\mathrm{pert}}(E_T - \delta E_T) \quad .$$

We can perform a Taylor expansion about E_T for small δE_T

$$\begin{aligned} f(E_T) &= f_{\mathrm{pert}}(E_T - \delta E_T) \\ &\simeq f_{\mathrm{pert}}(E_T) - \delta E_T \, \frac{df'_{\mathrm{pert}}(E_T)}{dE_T} \\ &= f_{\mathrm{pert}}(E_T) \left\{ 1 + n \, \frac{\delta E_T}{E_T} \right\} \quad . \end{aligned}$$

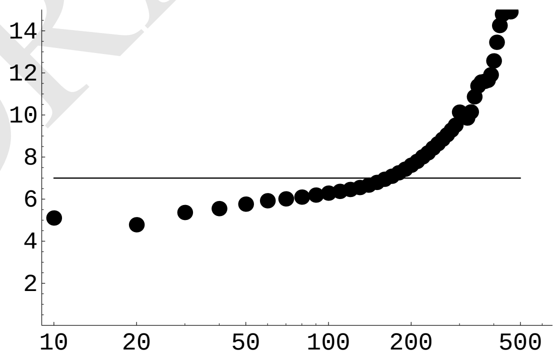

Figure 10: Power law behavior at the Tevatron, $n = -d\log(d\sigma/dE_T)/d\log(E_T)$ plotted versus E_T. A guideline is drawn at $n = 7$.

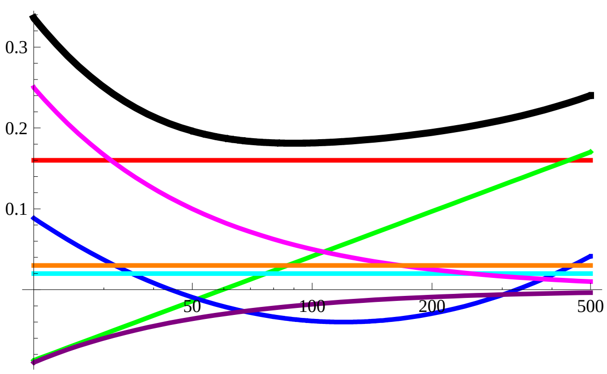

Figure 11: A compilation of the uncertainties for jet production. The upper thick line (black) is the quadrature sum of the individual errors.

Here we have used the power law of Eq. (12) to replace $df'(E_T) = -n\, f(E_T)/E_T$. Thus, to first order we find[3]

$$\frac{d\sigma}{dE_T} \simeq \frac{d\sigma_{\rm pert}}{dE_T}\left[1 + n\,\frac{\langle \delta E_T\rangle}{E_T} + ...\right] \qquad (13)$$

so that the fractional correction is $n\,\langle\delta E_T\rangle/E_T$. We use $n \approx 7$. Our estimate in Eq. (11) is that expected value of δE_T is about 0.5 GeV with an uncertainty of 0.7 GeV. We use the uncertainty to estimate an uncertainty in the cross section arising from underlying event and hadronization corrections. Taking 7×0.7 GeV ≈ 5 GeV, the uncertainty in the cross section from this source is

$$f_6(E_T) = \frac{5\ \text{GeV}}{E_T} \quad . \qquad (14)$$

VII. SUMMARY

We now summarize the contributions to the uncertainty of the jet E_T distributions:

$$\begin{aligned}
f_0(E_T) =& 0.16\\
f_1(E_T) =& 0.08\left[\log(15\,E_T/\sqrt{s}) + 0.7\right]\\
f_2(E_T) =& 0.04\left[\log(15\,E_T/\sqrt{s})^2 - 1.0\right]\\
f_3(E_T) =& 0.03\\
f_4(E_T) =& \frac{-0.001}{E_T/\sqrt{s}}\\
f_5(E_T) =& 0.02\\
f_6(E_T) =& \frac{5\ \text{GeV}}{E_T}
\end{aligned} \quad . \qquad (15)$$

These represent correlated errors. The net error at any one value of E_T is

$$\mathcal{E}(E_T) \equiv \sqrt{\sum f_i(E_T)^2} \quad . \qquad (16)$$

We display this in Fig. 11. For $E_T > 60$ GeV, the purely perturbative errors are dominant. The estimated error is about 20%, slowly rising with E_T as the needed partonic momentum fractions rise. For $E_T < 60$ GeV, the errors associated with non-perturbative, power suppressed, contributions become more important.

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